Exam

Autumn 2018

Important: Please make sure that you answer all questions and that you properly explain your answers. For each step write the general formula (where relevant) and explain what you do. Not only the numerical answer. If you make a calculation mistake in one of the earlier sub-questions, you can only get points for the following subquestions if the formula and the explanations are correct!

1. Short questions.

   (a) "The reason that players cannot achieve a good outcome in the prisoner's dilemma is that they cannot communicate." True or false? Explain in 2-3 sentences.

   > **Solution:** False, even if players could communicate their best response would still be to play defect.

   (b) "A simultaneous game can never be displayed as an extensive form game" True or false? Explain in 2-3 sentences.

   > **Solution:** False! But because it is simultaneous, the other player does not know where in the game he/ she is. It is a game of imperfect information and that needs to be reflected when writing it in extensive form.

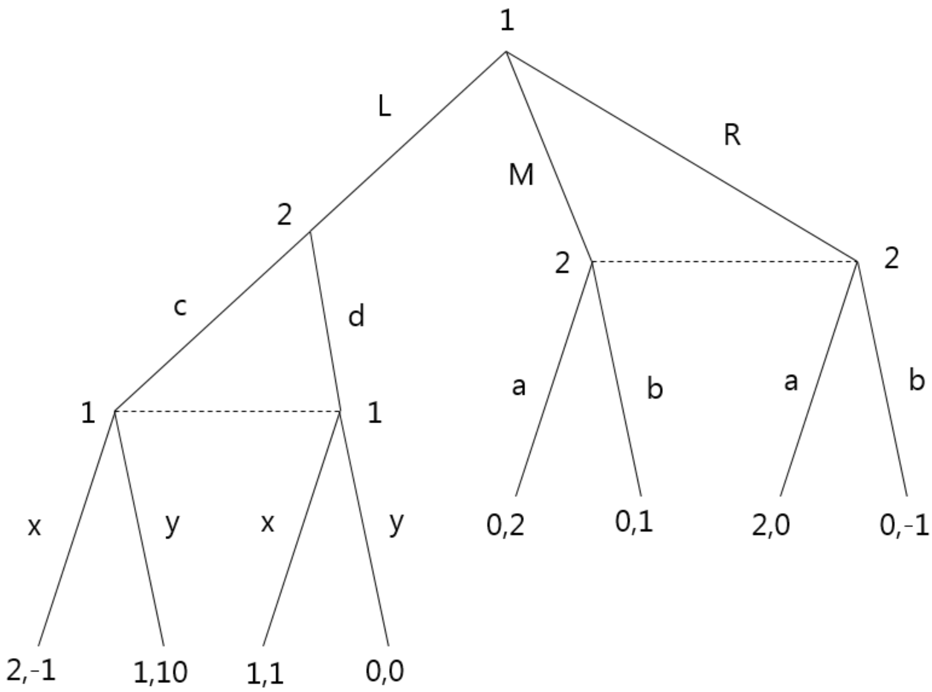
   (c) "Iterated Elimination of Strictly Dominated Strategies never eliminates a Nash Equilibrium" True or false? Explain in 2-3 sentences.

   > **Solution:** True! Nash equilibrium is a refinement. However, this does not hold for weakly dominated strategies.

   (d) You are writing your dating app profile and want to signal that you are adventurous. Give an example of a signal that is not credible and an example that is credible and explain the reasons why.

   > **Solution:** Credible signals: Show a picture of you skydiving, swimming with sharks etc. Not credible: Write that you are adventurous or only claim that you have been skydiving etc. There needs to be a differential cost that makes it affordable for those with a hidden desirable trait (being adventurous), not affordable for those without this trait.
   > There is a cost to the signal. Those with the desirable trait are more likely to send the signal. Those who exhibit the signal are more desirable.

2. Have a look at the game in Figure 1.



(a) Is it a dynamic or static game? How many proper subgames are there? What are the strategy sets of the players?

> **Solution:** Dynamic Game
> One proper subgame.
> Strategy sets Player 1: $S_1 = \{Lx, Ly, Mx, My, Rx, Ry\}$ Player 2: $S_2 = \{ca, cb, da, db\}$

(b) Find the pure strategy Nash Equilibria of each subgame.

> **Solution:** We need to find the SPNE of every subgame. We can start with the proper subgame. The SPNE of this subgame is (x,d). Having solved this subgame the whole game reduces to a game with payoff (1,1) after playing L. Then do the same for the whole game.

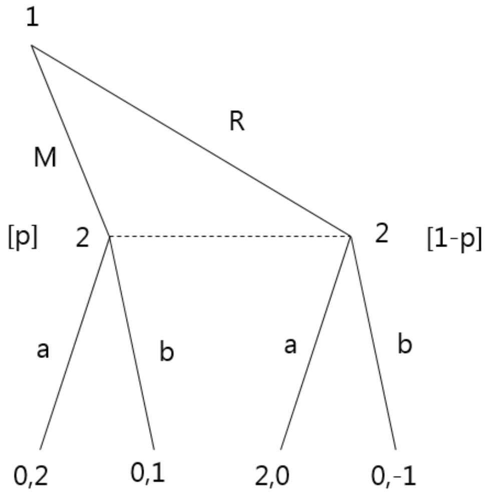
	a	b
L	$1, 1*$	$1*, 1*$
M	$0, 2*$	$0, 1$
R	$2*, 0*$	$0, -1$

> We find two NE, which are the SPNE we are looking for. (Lx, bd) and (Rx, ad).

(c) Is this a game of incomplete or imperfect information? Explain!

> **Solution:** It is a game of imperfect information. Players are simply unaware of the actions chosen by the others, however they know what the possible strategies/ actions are and the payoffs of the other players. Information is complete.

(d) Suppose that action "L", is no longer possible, and all nodes that follow action "L" are removed from the game. Now consider the game tree of this modified game. What information is missing from this game tree, that would need to be added, in order for us to solve for a Perfect Bayesian Equilibrium? Draw the game tree and add the missing information.

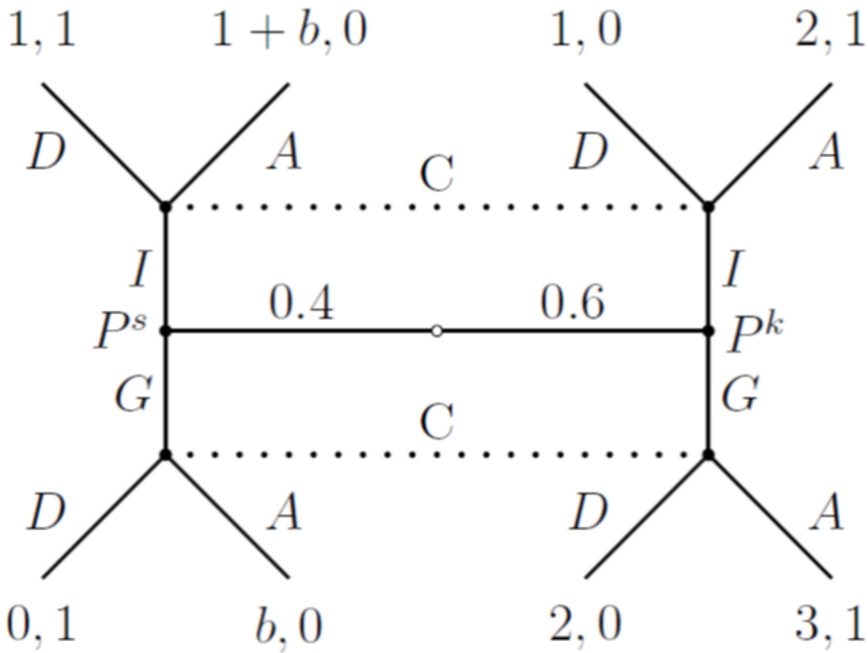


Solution: In a game of incomplete information Requirement 1 specifies that for a PBE players need to have beliefs about which node in the information set has been reached. For this we need to add a parameter for the belief p.

(e) What is the Perfect Baysian Equilibrium in this modified game?

Solution: The PBE is (R,a, p=0) or (R,a, p=1) if the game was labled the other way around.

3. Anders and Chris are walking down the street when a beggar approaches Anders and asks Anders for a dollar. Anders can either ignore (I) the beggar or give (G) the beggar a dollar. Chris knows that there are two types of people in this world, selfish and kind, and that there is only a 40% chance that a random person is selfish. Chris cannot directly observe Anders's type, so Chris draws inferences from Anders' behavior. A selfish Anders doesn't care at all about the beggar, so giving a dollar costs exactly a dollar's worth of utility. However, kind Anders is so kind that giving a dollar actually increases Anders utility by one. After observing Anders' choice, Chris expresses either disapproval (D) or approval (A). Chris wants to disapprove of the selfish Anders and approve of the kind Anders. Both types of Anders like Chris's approval, selfish Anders' approval can vary based on the situation, however, it is always positive. He gains $b > 0$ from it.



(a) Let $b = 0.5$ and find a separating PBE. What is the maximum value for b for which this separating equilibrium exists?

> **Solution:** The strategy profile is (IG, DA) where Anders' strategy first states the action of the selfish type and then of the kind type. Chris' strategy first states the response to I and then to G. The beliefs are $Pr(s|I) = 1$ and $Pr(s|G) = 0$. The equilibrium exists as long as $b \leq 1$.

(b) Let b = 100 and find a pooling PBE in which both types of Anders give. What is the smallest value of b for which this pooling equilibrium exists?

> **Solution:** The PBE is $[(GG, DA), Pr(s|I) \geq 1/2, Pr(s|G) = 0.4]$. This equilibrium exists as long as $b \geq 1$.

(c) Suppose kind people were more rare and selfish people more common, e.g., the initial probability that a random person is selfish is 0.6, instead of 0.4. State whether or not there can be a pooling equilibrium in which both types give and briefly explain your reasoning (without equations).

> **Solution:** If a random person is more likely than not to be selfish, there can be no pooling equilibrium in which both types give. Giving is uninformative when both types do it and, since selfish people are more likely than kind people, Chris's best response is to disapprove when Anders gives. The promise of approval is the only reason selfish Anders has to give, so selfish Anders would want to deviate and ignore the beggar.

4. The government decides to auction off the rights to drill all of the oil under Himmelbjerget. Mia and Peter decide to participate in the auction and, not knowing exactly how much oil is underground, each hires a consultant to estimate the size of the oil reserves. Consultants are expensive, though, and Mia and Peter can each only afford to pay the consultant to estimate the oil under one side of the mountain. Mia's consultant estimates that there are e_i dollars worth of oil under the north side of the mountain and Peter's consultant estimates

that there are e_j dollars worth of oil under the south side of the mountain, where e_i and e_j are both drawn uniformly from the interval [0, 1]. Thus, the total value of the oil is $v = e_i + e_j$, but because each person keeps her own estimate secret from the other, they each only know their own estimate. The only thing that each person knows about the other person's estimate is that it is drawn uniformly from the unit interval.

Suppose that the government holds a second-price auction for the oil rights. Show that there is a Baysian Nash Equilibrium in which each player bids twice her estimate. Verify this claim, step-by-step, by showing that if Peter's strategy is $b_j = 2e_j$, then Mia's best response is $b_i = 2e_i$.

(a) Write down the probability that Mia wins as a function of her bid, b_i (given Peter's strategy).

> **Solution:** $Pr(win) = Pr(b_j < b_i) = Pr(2e_j < b_i) = Pr(e_j < b_i/2) = b_i/2$

(b) Write down the expected price that Mia pays if she wins.

> **Solution:** Winning means that $b_j < b_i$, so the price would be b_j. The expectation of b_j given that $b_j < b_i is b_i/2$.

(c) Write down the expected value of the south side of the oil field, which Peter's consultant has estimated, if Mia wins.

> **Solution:** Because $b_j = 2e_j$, the expected value of e_j is $E[b_j/2|b_j < b_i]$, which can be rewritten as $1/2E[b_j|b_j < b_i]$. This is equal to 1/2 times the answer to part 2, so the answer is $b_i/4$.

(d) Using these three calculations, write down Mia's payoff for bidding b_i, given e_i is $= 1/2[e_i b_i - b_i^2/4]$.

> **Solution:** $EU_i(b_i|e_i) = Pr(win)E[payoff|win] + Pr(lose)E[payoff|lose]$.
> Because the payoff from losing is zero, this simplifies to
> $EU_i(b_i|e_i) = Pr(win)E[payoff|win]$.
> The payoff from winning is the value $v = e_i + e_j$ minus the price. Therefore we get:
> $EU_i(b_i|e_i) = b_i/2[e_i + b_i/4 - b_i/2] = 1/2[e_i b_i - b_i^2/4]$

(e) Show that the bid b_i that maximizes this expected payoff is $b_i = 2e_i$.

> **Solution:**
>
> $$\frac{\delta EU_i}{\delta b_i} = \frac{e_i}{2} - \frac{b_i}{4} = 0$$
>
> The solution to this first-order condition is $b_i^* = 2e_i$, so the strategy is a best response as claimed.